

# Sgr A\*: The Optimal Testbed of Strong-Field Gravity

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**Abstract.** The black hole in the center of the Milky Way has been observed and modeled intensely during the last decades. It is also the prime target of a number of new experiments that aim to zoom into the vicinity of its horizon and reveal the inner working of its spacetime. In this review we discuss our current understanding of the gravitational field of Sgr A\* and the prospects of testing the Kerr nature of its spacetime via imaging, astrometric, and timing observations.

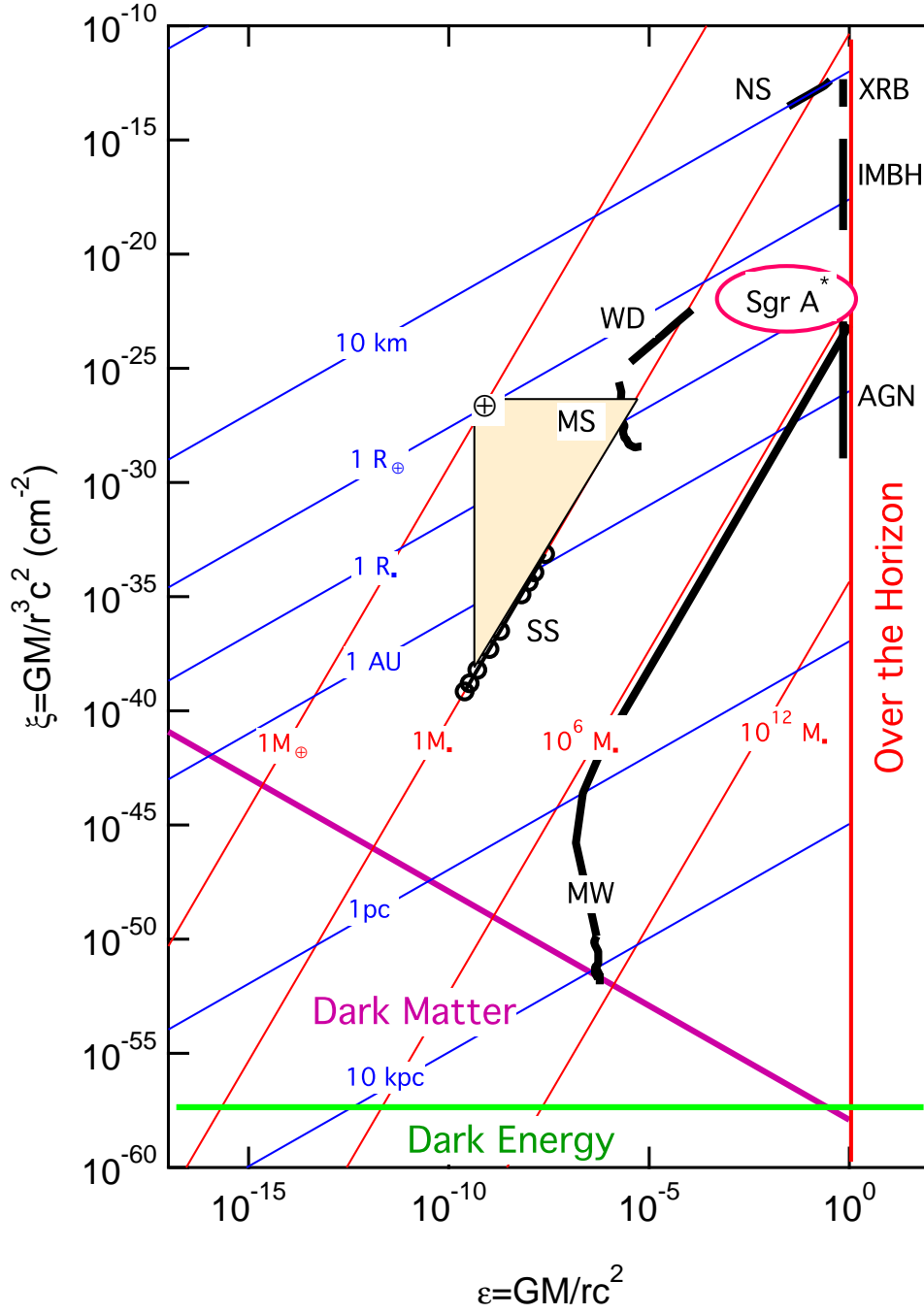
## 1. Introduction

The Kerr spacetime of spinning black holes is one of the most unexpected predictions of Einstein's theory of General Relativity. The special role this spacetime plays in the theory of gravity is encapsulated in the no-hair theorem, which states that the Kerr metric is the only stationary, axisymmetric, asymptotically flat solution to the vacuum field equations that possesses a horizon but no closed timelike loops (Israel 1967, 1968; Carter 1971, 1973; Hawking 1972; Robinson 1975; Mazur 1982). Because of this theorem, it is expected that all astrophysical objects that have been identified as black-hole candidates are indeed described by the Kerr metric.

The Kerr nature of astrophysical black holes is a prediction that can be tested observationally. Doing so offers the unique opportunity of both rejecting alternative interpretations of their nature and of verifying General Relativity in the strong-field regime. There are at least three astrophysical settings we know of today, in which black holes appear to exist: in galactic X-ray binary systems, as ultraluminous X-ray sources, and in active galactic nuclei. Among all these black holes, the one in the center of our galaxy, Sgr A\*, combines a large mass with a small distance from the sun (e.g., Ghez et al. 2008; Gillessen et al. 2009). It can also be probed by a variety of observations throughout the electromagnetic spectrum making it the optimal test bed of the Kerr metric.

Recent and anticipated advances in observations of Sgr A\* throughout the electromagnetic spectrum have both secured our understanding of the basic properties of this black hole (e.g., Reid 2009), and opened new opportunities for devising tests of gravity theories. At the same time, there have been significant recent advances in the development of a theoretical framework with which observations of black holes can be used to test quantitatively the Kerr metric and search for violations of the no-hair theorem.

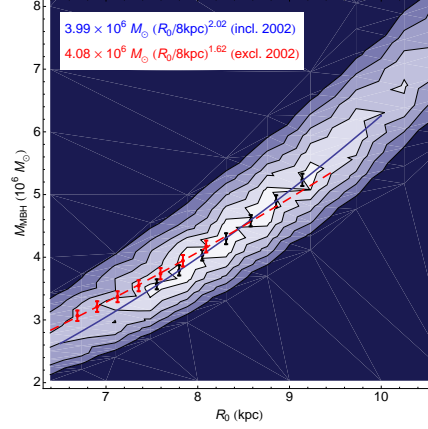
In this article, we review our current understanding of the properties of the black hole in the center of the Milky Way and discuss the prospect of testing the no-hair theorem with upcoming observations.



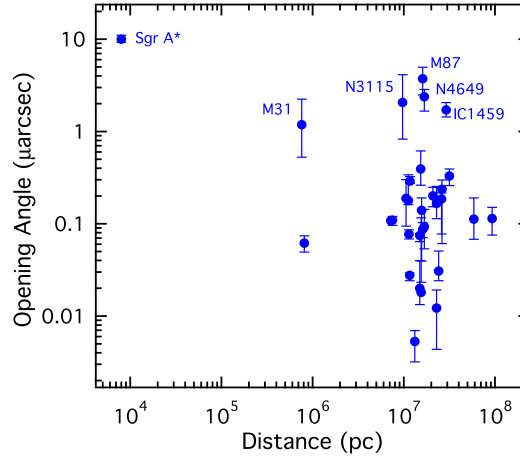
**Figure 1.** A parameter space for tests of General Relativity. The  $x$ -axis measures the potential and the  $y$ -axis the curvature of the field probed by different experiments. A number of astrophysical and cosmological objects are also shown. The black hole in the center of the Milky Way, Sgr A\*, probes a region of the parameter space that has not been investigated with other current tests; the location of the latter is outlined by the inverted yellow triangle in the center of the figure (Psaltis 2008).

## 2. Sgr A\*: The Optimal Black Hole for Tests of the No-Hair Theorem

The black hole in the center of the Milky Way is the optimal astrophysical object for testing our understanding of black hole spacetimes, for a number of reasons that we outline below.

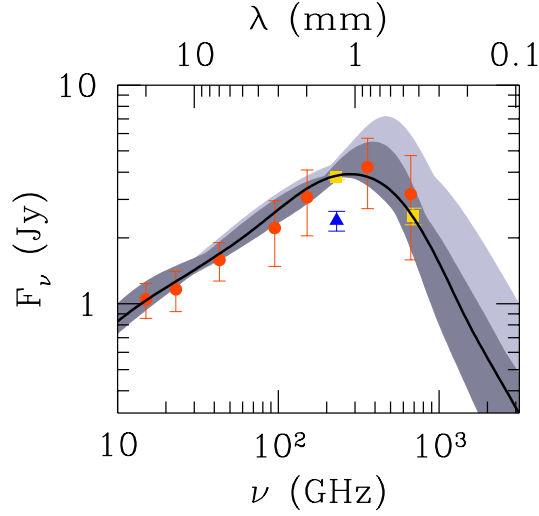


**Figure 2.** Confidence contours of the mass and distance to Sgr A\*, based on the analysis of orbits of nearby stars (Gillessen et al. 2009).



**Figure 3.** The opening angles, as viewed by an observer on Earth, of the horizons of a number of supermassive black holes in distant galaxies with a secure dynamical mass measurement (sample of Tremaine et al. 2002). The opening angle of the black hole horizon in the center of the Milky Way (Sgr A\*) is also shown for comparison (after Psaltis 2008).

- **Sgr A\* probes a new regime of gravitational field strengths.** General Relativity has been tested in a rather limited range of gravitational field strengths (Fig. 1). Phenomena in the vicinity of the black-hole horizon around Sgr A\* probe gravitational fields with a potential and a curvature that are five orders of magnitude larger than the strongest fields probed by current tests of General Relativity.
- **Sgr A\* has an accurately measured mass.** Fitting the trajectories of stars in the vicinity of Sgr A\* has lead to an accurate measurement of its mass. A recent careful analysis of the orbits by Gillessen et al. (2009; see also Ghez et al. 2008) resulted in a measurement of  $M = (4.31 \pm 0.06 \pm 0.36) \times 10^6 M_\odot$  with the second uncertainty corresponding to the error in the estimate of the distance to the center of the Milky Way (see Fig. 2). This highly accurate mass measurement specifies one of the only two basic parameters of the black hole and sets uniquely the scale for any anticipated relativistic phenomena.
- **The mass measurement provides an independent distance measurement to Sgr A\*.** Testing the Kerr spacetime via, e.g., analysis of the images of its inner accretion



**Figure 4.** The long-wavelength spectrum of Sgr A\*. The grey filled areas show the envelopes of semi-analytic models that best match the data (Broderick et al. 2009).

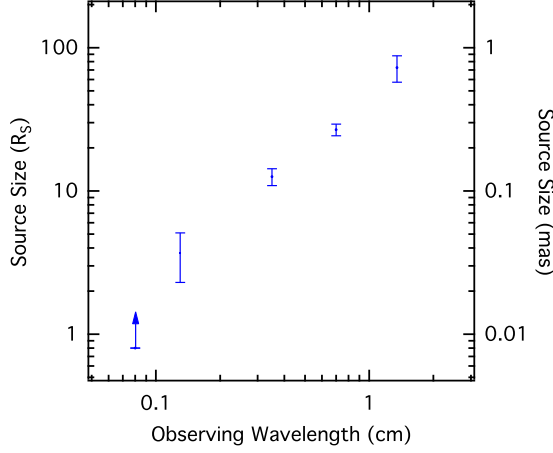
flow, requires an independent knowledge of the distance to the source in order to convert angular separations to physical lengths. Studies of the orbits of stars that approach Sgr A\* lead to a measurement of the distance to the source of  $R_0 = 8.33 \pm 0.35$  kpc (Gillessen et al. 2009).

- **Sgr A\* is one of the only two black-holes with horizons that will be resolved with sub-mm VLBI in the very near future.** For a supermassive black hole in a distant galaxy, the opening angle of the horizon as viewed from the earth is

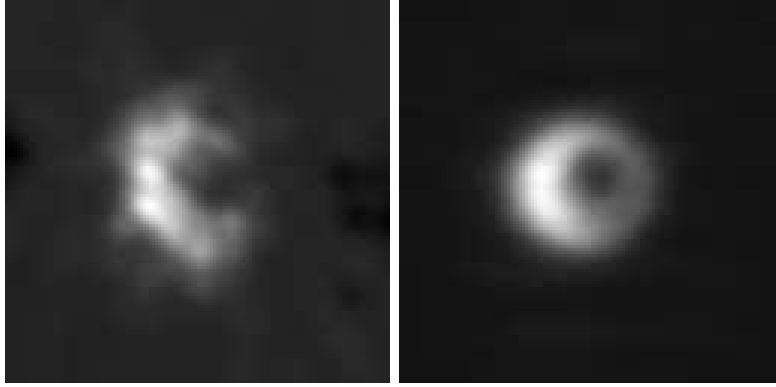
$$\theta = 20 \left( \frac{M}{10^9 M_\odot} \right) \left( \frac{1 \text{ Mpc}}{D} \right) \mu\text{arcsec} . \quad (1)$$

This is shown in Figure 3 for a number of supermassive black holes with secure mass determinations. Sgr A\* is the black hole that combines the highest brightness with the largest angular size of the horizon.

- **At photon frequencies  $> 5 \times 10^{11}$  Hz, the emission from Sgr A\* is optically thin.** The long-wavelength spectrum of Sgr A\* peaks at a frequency of  $\simeq 5 \times 10^{11}$  Hz, suggesting that the emission changes from optically thick (probably synchrotron emission) to optically thin at a comparable frequency (see Fig. 4). As a result, observations at frequencies comparable to or higher than this transition frequency are expected to probe the region close to the black-hole horizon without significant obscuration. This fact is indeed confirmed by the measurement of the size of Sgr A\* at this part of the spectrum (see below).
- **The size of the emitting region in Sgr A\* at long wavelengths is comparable to the horizon scale.** Since the first measurements of the size of the source at 7 mm and at 1.4 mm (Krichbaum et al. 1998) demonstrated that the emitting region is only a few times larger than the radius of the horizon (see Figure 5), a number of observational investigations have aimed to probe deeper into the gravitational field of the black hole. Most recently, observations at 1.3 mm (Doeleman et al. 2008) revealed near-horizon sizes for the emitting region of Sgr A\*.
- **The detailed structure of the inner accretion flow around Sgr A\* will be imaged in the very near future.** Figure 6 shows two simulated images of Sgr A\* at 345 GHz



**Figure 5.** The major axis of the accretion flow around the black hole in the center of the Milky Way, as measured at different wavelengths, in units of the Schwarzschild radius (left axis) and in milliarcsec (right axis; adapted from Shen et al. 2005 and Doeleman et al. 2008; after Psaltis 2008). Even with current technology, the innermost radii of the accretion flow can be readily observed.



**Figure 6.** Simulated images of the inner accretion flow around Sgr A\*, as they will be resolved with (*left*) a 7-telescope array in the near future (3-5 years) and (*right*) a 13-telescope array (Fish & Doeleman 2009).

as they will be resolved by a 7-telescope array in the near future (3-5 years) and by a 13-telescope array further in the future (Fish & Doeleman 2009). The characteristic asymmetry in the image caused by Lorentz boosting as well as the shadow of the black hole are clearly visible.

### 3. Parametrizing Deviations from the Kerr Metric

The Kerr metric, which describes the exterior spacetime of a spinning black hole, is not unique to General Relativity. Indeed, a large number of simple extensions to the general relativistic field equations are also satisfied by the Kerr solution (Psaltis et al. 2008). This is perhaps not surprising (see, e.g., Barausse & Sotiriou 2008), given the fact that the Kerr metric is derived by solving the vacuum field equation

$$R_{\mu\nu} = 0, \quad (2)$$

where  $R_{\mu\nu}$  is the Ricci tensor, and not the full Einstein field equation (the two are equivalent in vacuum). Equation (2), however, is also satisfied by the Minkowski metric of special relativity. As a result, the field equations of any theory that has special relativity as its vacuum limit should

also be equivalent in vacuum to equation (2) and, therefore, be satisfied by the Kerr metric. This argument remains valid even when we allow for the possibility of a finite cosmological constant (see Psaltis et al. 2008).

Constructing a modification to General Relativity in which the black hole solutions are not described by the Kerr metric can be done, in principle, by violating one of the more fundamental assumptions in our understanding of gravity, such as the equivalence principle (see, e.g., Eling & Jacobson 2006). The only black-hole solution in a theory that obeys the equivalence principle that we are aware of is that of Yunes & Pretorius (2009) and Konno et al. (2009) in Chern-Simons gravity. In Chern-Simons gravity, a parity violating term is added to the Einstein-Hilbert action of General Relativity, with a coupling constant that is a dynamical field (see Alexander & Yunes 2009 for a review). This parity violating term is zero for a parity-symmetric metric and, therefore, the Minkowski and Schwarzschild spacetimes remain solutions to the field equations. However, the spacetime of a spinning black hole has a particular parity specified by its spin and, therefore, it is different than the Kerr metric.

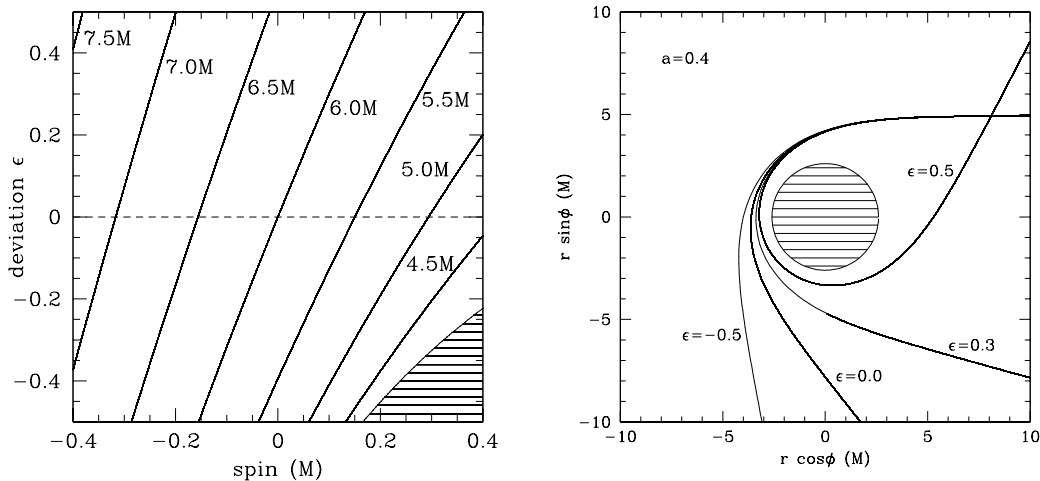
Given the very limited range of available black-hole metrics in modified gravity theories that is known today, it is more fruitful to test the Kerr metric in a phenomenological way, by adding parametrically terms to its various elements (see Psaltis 2009 for a discussion). This is equivalent to testing for violations of the no-hair theorem, even within General Relativity.

The no-hair theorem states that the only stationary, axisymmetric, asymptotically flat vacuum solution to the Einstein field equations that possesses a horizon and no time-like loops is the Kerr metric (we do not consider here the unlikely possibility that an astrophysical black hole will have a net charge). The Kerr metric is uniquely determined by only two parameters, the mass and the spin of the black hole. This allows us to define a formal test of the no-hair theorem, based on the work of Ryan (1995), in the following way (see also Collins & Hughes 2004; Glampedakis & Babak 2006; Vigeland & Hughes 2010; Vigeland 2010).

We can, in principle, expand the exterior metric of any compact object in multipoles and use observations to measure the coefficients of the expansion. Because of the no-hair theorem, only two of the multipole coefficients for the spacetime of a black hole are independent. The coefficient of the monopole is the mass  $M$  of the black hole and of the dipole is its spin  $a$ . All higher-order coefficients will depend on the first two, in the particular way dictated by the Kerr metric. Testing the no-hair theorem requires measuring at least the coefficient of the quadrupole  $q$  and verifying whether it satisfies the Kerr relation  $q = -a^2$ .

Four different approaches have been explored so far for introducing additional non-Kerr hair to the spacetimes of compact objects. Ryan (1995) studied a general expansion of stationary, axisymmetric, asymptotically flat spacetimes in Geroch-Hansen multipoles. Collins & Hughes (2004) as well as Vigeland & Hughes (2010) added Weyl-sector bumps to the Schwarzschild and Kerr spacetimes. Glampedakis & Babak (2006) modified the Kerr metric by adding an arbitrary quadrupole moment while ensuring that the metric to that order remains a solution to the vacuum field equation (2). Finally, Gair et al. (2008) explored in detail the parametric solution of Manko & Novikov (1992), which allows for deviations of all higher-order multipoles from their Kerr values. All these approaches were developed originally in order to test General Relativity with future observations of the gravitational waves generated during inspirals into supermassive black holes. However, they are also directly applicable to tests of gravity with Sgr A\*.

In Johannsen & Psaltis (2010a, 2010b, 2010c), we focused on the parametric post-Kerr spacetime obtained by Glampedakis & Babak (2006) for two reasons. First, this approach uses a single parameter associated to the quadrupole moment of the spacetime to quantify potential deviations from the Kerr metric, making it the simplest and most concise possible avenue to testing the no-hair theorem. Second, the complete metric of Glampedakis & Babak (2006) remains a valid solution to the vacuum Einstein field equations, allowing us to perform



**Figure 7.** The dependence of (*Left*) the radius of the innermost stable circular orbit around a black hole and of (*Right*) the trajectories of photons on the black-hole spin  $a$  and on the parameter  $\epsilon$  that measures the degree of violation of the no-hair theorem (Johannsen & Psaltis 2010a, 2010c).

a self-consistent test of the no hair theorem and of the black-hole identification of the compact object, within General Relativity.

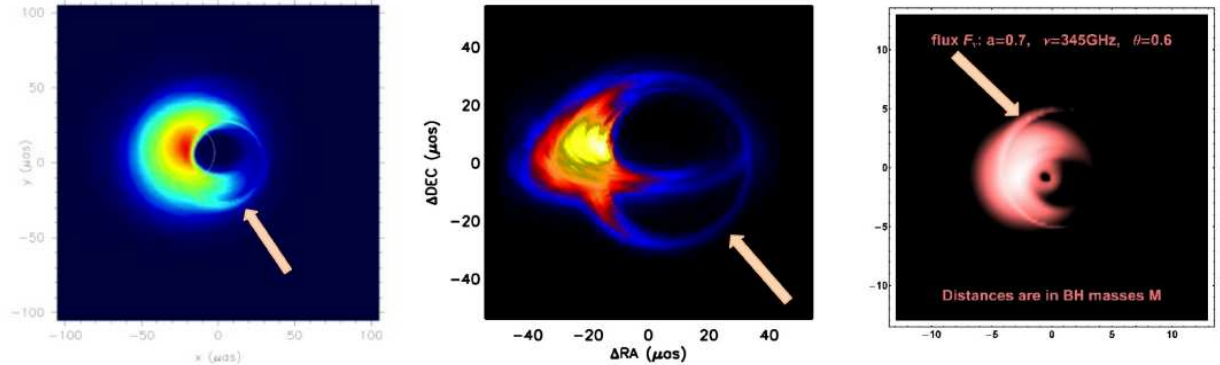
The key drawback of all currently proposed parametric deviations from the Kerr metric is that they cannot be used in describing the exterior spacetimes of rapidly spinning black holes. Precisely because of the no-hair theorem, it is impossible to construct a spacetime that deviates from the Kerr metric, is a solution to the general relativistic vacuum field equations, and is regular everywhere outside the black-hole horizon; such a construction would prove that the no-hair theorem is violated. In all parametric deviations from the Kerr metric, the spacetimes become irregular at radii  $\simeq 2M$  (see Gair et al. 2008; Johannsen & Psaltis 2010a). For moderate black-hole spins, radii comparable to  $2M$  are much smaller than the radius of the photon orbit and of the innermost stable circular orbit. As a result, such radii can be artificially excised, without affecting the prediction of any observable. For rapidly spinning black holes, however, all characteristic radii become  $\leq 2M$  and, therefore, the irregularities of the spacetimes preclude us from calculating various observables related, e.g., to the inner accretion flow around a black hole. We do not anticipate this to introduce significant problems for the case of Sgr A\*, because preliminary attempts to model the VLBI images from the source show preference for small values of its spin ( $a \leq 0.3$ ; Broderick et al. 2009, 2010).

#### 4. Testing the No-Hair Theorem with Sgr A\*

In Johannsen & Psaltis (2010a), we explored in detail the properties of the Glampedakis & Babak (2006) metric, addressing its potential in testing the no-hair theorem with astrophysical observations in the electromagnetic spectrum. Following their original analysis, we expressed the coefficient of the quadrupole multipole of the spacetime as

$$q = -(a^2 + \epsilon) \quad (3)$$

with the parameter  $\epsilon$  measuring the degree of violation of the no-hair theorem. We then studied the trajectories of photons and particles in this spacetime and identified three important effects of the presence of a non-Kerr quadrupole.



**Figure 8.** The presence of a bright photon ring surrounding the black-hole shadow is ubiquitous in all current simulations of images from radiatively inefficient accretion flows (from left: Moscibrodzka et al. 2009; Dexter et al. 2009; Shcherbakov & Penna 2010). The size, location, and shape of the photon ring is determined by the projected photon orbit at infinity and, as such, it depends only on the metric of the spacetime. The accretion flow is necessary only to provide the photons that will trace the photon ring for an observer at infinity.

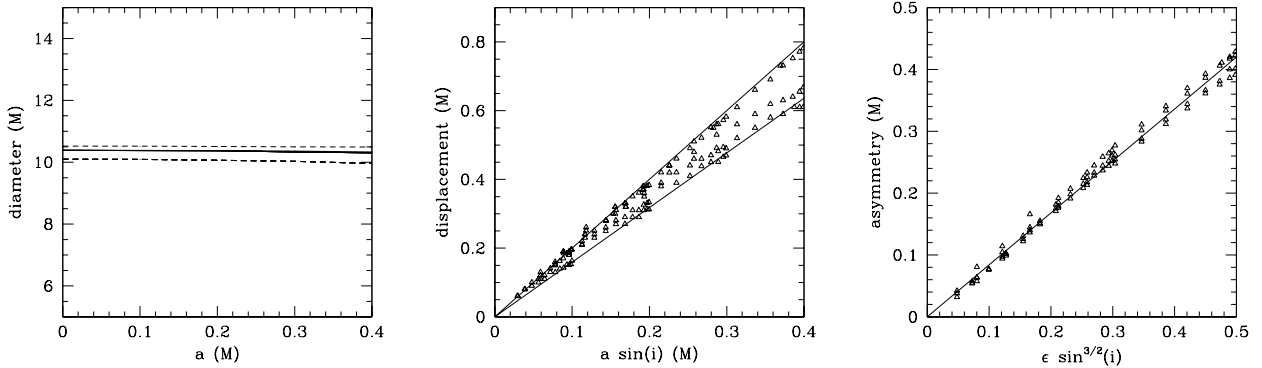
First, the location of the innermost stable circular orbit (ISCO) is significantly altered, as shown in Figure 7 (left panel). The location of the maximum emission from the accretion flow around Sgr A\* is expected to be very close to that of the ISCO (see, e.g., Krolik & Hawley 2002 and references therein) and, therefore, the brightness profile of the image from Sgr A\* will depend on the value of the quadrupole. Second, the radius of the photon orbit is also significantly affected by the value of the quadrupole. The radius of the photon orbit determines the size of the shadow of the black hole (see Bardeen 1973; Falcke et al. 2000), and generates one of the most prominent features in the image of Sgr A\* (i.e., the structure that we call the photon ring; see below). Finally, the detailed trajectories of photons that propagate close to the photon orbit depend also on the magnitude of the quadrupole (see Fig. 7, right panel) leading to non-trivial deformations of the calculated images.

Figure 7 also shows, that changing the quadrupole moment of a black-hole spacetime may even alter qualitatively the properties of particle and photon orbits in the vicinity of its horizon. In the Glampedakis & Babak (2006) metric, for each value of the spin, there exists a minimum quadrupole deviation beyond which all circular orbits are stable to radial perturbations (Johannsen & Psaltis 2010c; this results has also been discussed for the Manko & Novikov metric by Gair et al. 2008). For relatively large black-hole spins ( $a > 0.4$ ), any reduction of the quadrupole moment of the spacetime from its Kerr value stabilizes all circular orbits against radial perturbations. In those spacetimes, equatorial particle orbits close to the black-hole horizon become instead unstable to vertical perturbations (see Gair et al. 2008; Johannsen & Psaltis 2010c) with implications for the inner accretion flows that have not been explored yet.

In Johannsen & Psaltis (2010b) we investigated the prospect of measuring the parameter  $\epsilon$  for Sgr A\* via imaging observations in the sub-mm. We identified in the simulated images of the accretion flow a bright ring (which we called the photon ring) that surrounds the black-hole horizon (see also, e.g., Beckwith & Done 2005). This ring is produced by photon rays that orbit the black hole multiple times at the radius of the photon orbit before escaping to infinity. Its presence is ubiquitous in all current simulations of images from radiatively inefficient accretion flows (see Fig. 8).

The location, size, and geometry of the photon ring corresponds to the projection of the photon orbit on the image plane of the distant observer. Because of this, its geometric





**Figure 9.** Three panels showing the (*Left*) diameter, (*Middle*) displacement, and (*Right*) asymmetry of the bright photon ring surrounding the black-hole shadow in the image of an accreting black hole, as a function of the parameters of the quasi-Kerr metric. The three depicted properties of the ring measure three different multipoles of the spacetime (modulo the sine of the observer’s inclination  $\sin i$ ) and can lead to a test of the no-hair theorem (Johannsen & Psaltis 2010b).

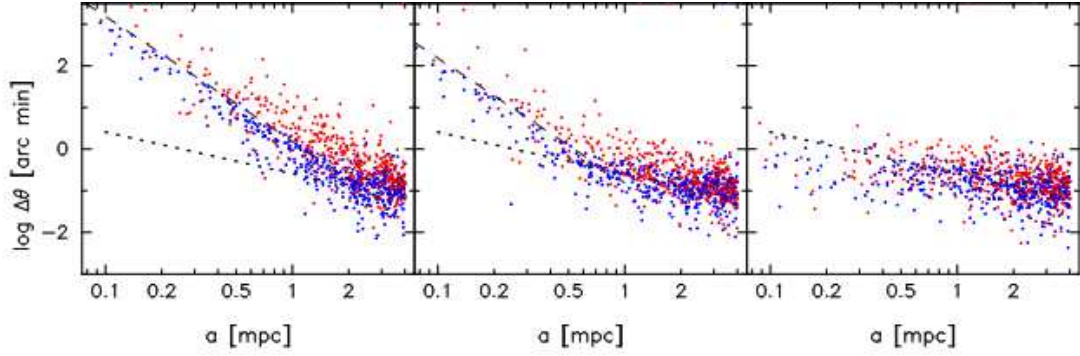
characteristics are independent of the particular accretion flow model assumed. The accretion flow is necessary only to provide the photons that will trace the photon ring for an observer at infinity. The diameter of the bright photon ring is  $\sim 10.0 - 10.5 M$ , practically independent of the spin and the quadrupole moment of the black hole, providing a direct measure of the black-hole mass (Figure 9). The displacement of the ring from the center of mass of the system depends primarily on the spin of the black hole. Finally, the deviation of the shape of the photon ring from a circle is a direct measure of the violation of the no-hair theorem (modulo the observer’s inclination). Indeed, photon rings for the Kerr metric remain practically circular for all but the fastest spins ( $a \leq 0.9$ ), whereas even a small degree of violation of the no-hair theorem (as measured by the parameter  $\epsilon$ ) introduces a significant asymmetry to the photon ring (see Figure 9). Imaging the inner accretion flow and measuring the asymmetry of the photon ring offers, therefore, the possibility of a clean, quantitative test of the no-hair theorem.

## 5. Distinguishing Gravitational Effects from Astrophysical Complications

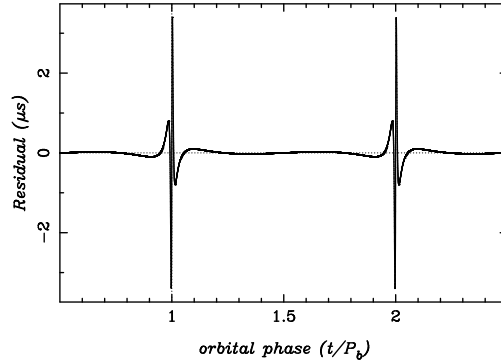
The biggest challenge in searching for violations of known physics with astrophysical measurements is ensuring that a particular measurement is not affected adversely by astrophysical complications. In the previous section, we identified a signature of gravitational effects that probes cleanly the metric of the black hole and is affected only marginally by other astrophysics, i.e., the bright photon ring that surrounds the shadow of the black hole. The location, size, and shape of the photon ring depends predominantly on the spin and quadrupole of the spacetime and only marginally on the properties of the underlying accretion flow. Even in this case, however, the narrow width of the ring (of order a few tens of  $M$ ), as well as possible confusion with other emission from the accretion flow between the observer and the horizon may make such a measurement biased.

We can convert a compelling argument for possible violations of the no-hair theorem into a bullet proof result by using different observational probes for the spin and quadrupole moment of the same black hole and by testing whether all such probes agree quantitatively. Sgr A\* is unique in that it potentially offers two additional, independent ways of testing the no-hair theorem.

Will (2008) showed that astrometric observations of the orbits of stars clustered very close to the black hole can be used to measure the quadrupole moment of the black hole via detection



**Figure 10.** The cumulative change in the angle  $\delta\theta$  between the initial and final orbital angular momenta of stars around Sgr A\* with different orbital semi-major axes  $a$ . The dashed lines show the effect of relativistic frame-dragging while the dotted lines correspond to an approximate model of precession due to stellar perturbations. The three panels show the result for black hole spins (left)  $a = 1$ , (middle)  $a = 0.1$ , and (right)  $a = 0$  (Merritt et al. 2010).



**Figure 11.** Typical residuals in the timing solution for a radio pulsar orbiting a black hole, when the effects of the quadrupole moment of the black hole spacetime have not been taken into account (Wex & Kopeikin 1999). The residuals shown here have been calculated for a  $10^4 M_\odot$  black hole; they will be similar in shape but larger in magnitude for a pulsar orbiting Sgr A\*.

of frame dragging. In a follow-up study, Merritt et al. (2010) showed that interactions between the stars of the cluster will mask frame dragging effects, unless the analysis is confined to stars within  $\simeq 1000$  Schwarzschild radii from the horizon (see Fig. 10). If such stars exist, future instruments (such as GRAVITY; Bartko et al. 2009) will be able to provide an independent measurement of the quadrupole moment of the black-hole spacetime.

The presence of a cluster of massive stars around Sgr A\* makes it practically certain that a number of radio pulsars will also be orbiting the black hole. Current surveys put an upper limit of as many as 90 normal pulsars within the central parsec of the Galaxy (e.g., Macquart et al. 2010). Detecting such a pulsar in one of the current surveys will allow us to measure the quadrupole moment of the black hole by looking for particular spin-orbit coupling residuals in the timing solution for each pulsar (see Fig. 11; Wex & Kopeikin 1999).

The combination of observations of the images from the inner accretion flow around Sgr A\*, of the relativistic precession of massive stars orbiting close to the black hole, and of the residuals in the timing solution of radio pulsars in similar orbits will allow for three, independent measurements of the spin and of the quadrupole moment of the black hole. Each observation will

be performed at different wavelengths and with different telescopes. Moreover, each of the three observed phenomena will probe different regions of the black-hole spacetime and will be affected by different systematics. If all three measurements give consistent solutions for the black-hole spin and quadrupole moment, then this will lead to an unequivocal test of the no-hair theorem with an astrophysical black hole.

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